

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Simplify $(k^3)^4$

$$k^{3 \times 4}$$

$$k^{12}$$

(1)

(b) Simplify $y^6 \times y^9$

$$y^{6+9}$$

$$y^{15}$$

(1)

(c) Expand $5m^2(m^2 + 2m)$

$$5m^2 \times m^2 = 5m^4$$

$$5m^2 \times 2m = 10m^3$$

$$5m^4 + 10m^3$$

(2)

(Total for Question 1 is 4 marks)

2 Jenny wants to know how many sandwiches she will need for 550 people at a meeting.

Each person who eats sandwiches will eat 3 sandwiches.

2 slices of bread are needed for each sandwich.

Jenny assumes 76% of the people will eat sandwiches. *

(a) Using this assumption, work out the number of slices of bread Jenny needs.

Give your answer correct to the nearest hundred slices.

$$\begin{aligned} 76\% \text{ of } 550 &= 0.76 \times 550 \\ &= 418 \end{aligned}$$

$$418 \times 3 = 1254 \text{ sandwiches}$$

$$1254 \times 2 = 2508 \text{ slices of bread.}$$

$$\approx 2500 \text{ slices}$$

..... 2500 slices

(4)

Jenny's assumption is wrong.

68% of the people will eat sandwiches.

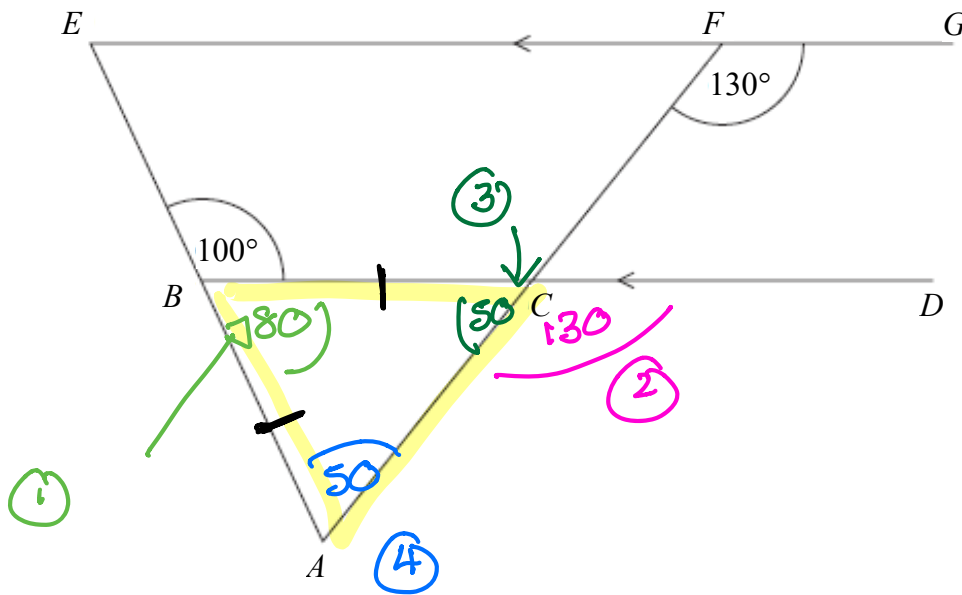
(b) How does this affect your answer to part (a)?

..... My answer would be less, so fewer
..... slices of bread would be needed.

(1)

(Total for Question 2 is 5 marks)

- 3 *ACF* and *ABE* are straight lines.
EFG and *BCD* are parallel lines.



Show that triangle *ABC* is isosceles.
 Give a reason for each stage of your working.

① angles on a straight line add up to 180°
 $180 - 100 = 80$

② corresponding angles are equal

③ angles on a straight line add up to 180
 $180 - 130 = 50$

④ $\angle BAC = 180 - (80 + 50) = 50$ Angles in a triangle add up to 180°

TRIANGLE *ABC* IS ISOSCELES AS IT HAS TWO EQUAL ANGLES.

(Total for Question 3 is 5 marks)

5 P and Q are numbers such that

$$P = 2^3 \times 3^5 \times 5$$

$$Q = 3^2 \times 5^3$$

(a) Find the highest common factor (HCF) of P and Q .

$$P = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5$$
$$Q = \quad \quad \quad 3 \times 3 \quad \quad \quad \times 5 \times 5 \times 5$$

$$\text{HCF} = 3 \times 3 \times 3$$
$$= 45$$

45

(1)

(b) Find the lowest common multiple (LCM) of P and Q .

$$\text{LCM} = 45 \times 2^3 \times 3^3 \times 5^2$$
$$= 243000$$

243000

(2)

(Total for Question 5 is 3 marks)

6 Sludge leaks from a pipe at a constant rate of $8.7 \text{ m}^3/\text{s}$

How many hours does it take for $98\,310 \text{ m}^3$ of sludge to leak from the pipe?

Give your answer correct to the nearest hour. = $60 \times 60 = 3600 \text{ seconds}$

$$8.7 \text{ m}^3 = 1 \text{ second}$$

$$\text{so } \frac{98310 \text{ m}^3}{8.7 \text{ m}^3} = 11300 \text{ seconds}$$

$$11300 \div 3600$$

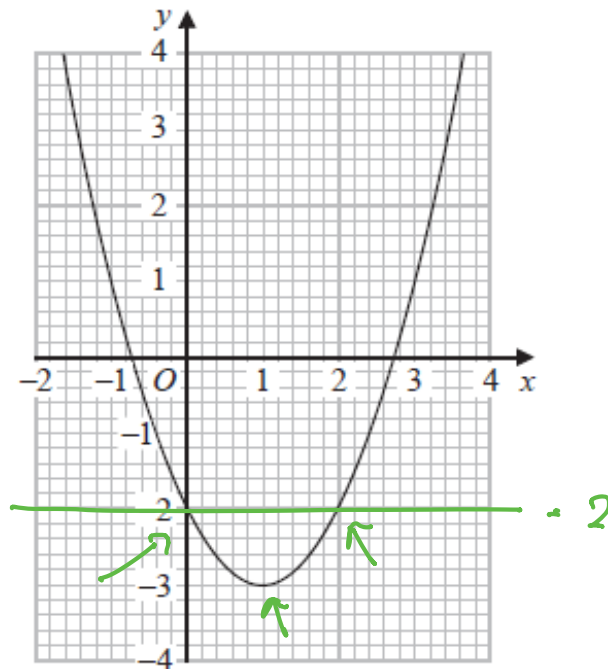
$$= 3.138$$

$$\text{so } 3$$

.....3..... hours

(Total for Question 6 is 3 marks)

7 Here is the graph of $y = x^2 - 2x - 2$



(a) Write down the coordinates of the turning point on the graph of $y = x^2 - 2x - 2$

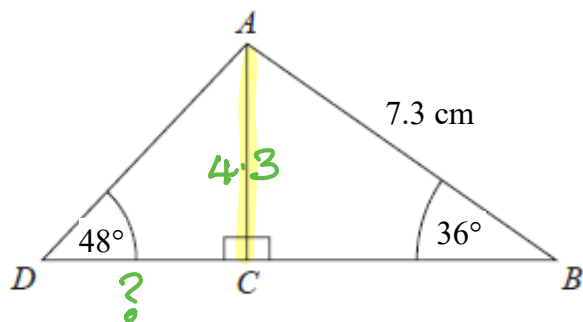
(..... 1, -3)
(1)

(b) Write down an estimate for one of the roots of $x^2 - 2x - 2 = -2$

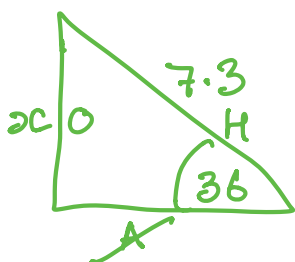
..... 0 or 2
(1)

(Total for Question 7 is 2 marks)

- 10 ABD is a triangle.
 C is a point on BD .



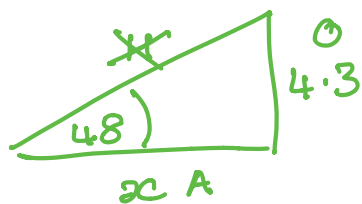
Work out the length of DC .
 Give your answer correct to 1 decimal place.



$$\sin 36 = \frac{x}{7.3}$$

$$x = 7.3 \times \sin 36$$

$$= 4.2938\dots$$



$$\tan 48 = \frac{4.29\dots}{x}$$

$$x = \frac{4.29\dots}{\tan 48}$$

$$= 3.86348\dots$$

$$= 3.9$$

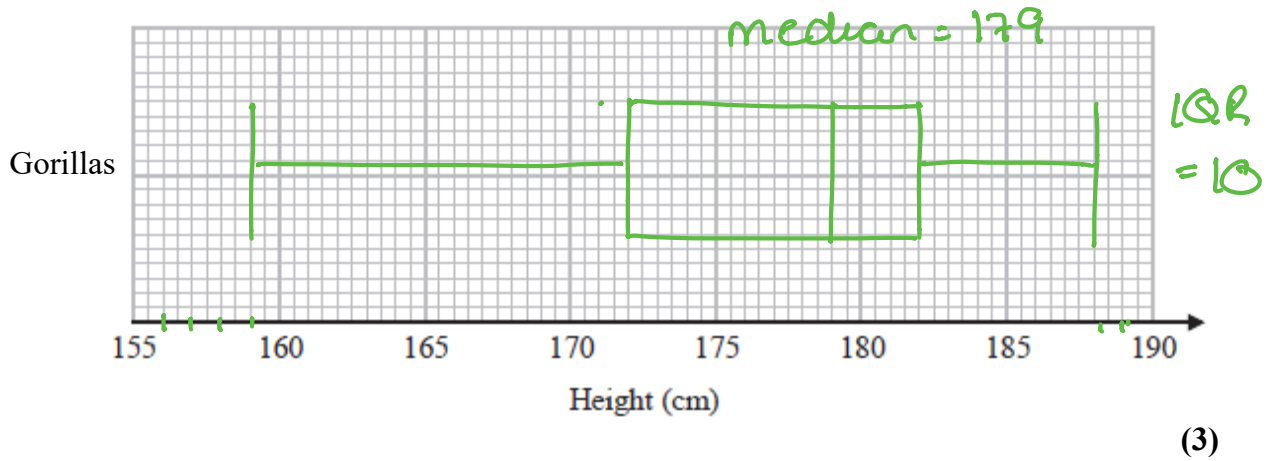
..... 3.9 cm

(Total for Question 10 is 3 marks)

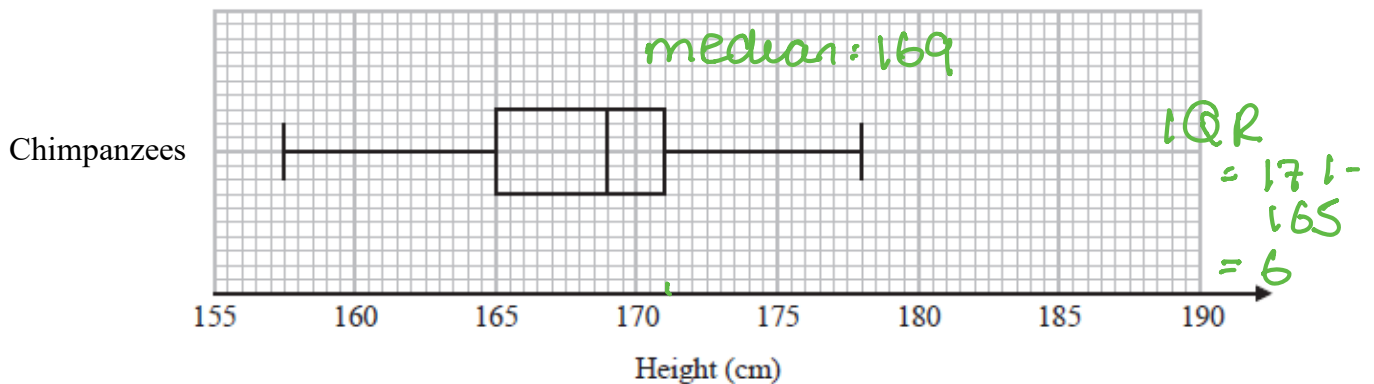
11 The table shows some information about the heights of a group of gorillas.

least height	159 cm ✓
greatest height	188 cm ✓
median	179 cm ✓
lower quartile	172 cm ✓
upper quartile	182 cm ✓

(a) On the grid, draw a box plot for the information in the table.



The box plot below shows the distribution of the heights of a group of chimpanzees.



(b) Compare the distribution of the heights of the gorillas with the distribution of the heights of the chimpanzees.

The median height of the gorillas is greater, so on average, they are taller.
 The IQR of the chimpanzees is smaller so their heights are more consistent.

(2)

(Total for Question 11 is 5 marks)

- 12 Show that $(x+2)(x-3)(x+4)$ can be written in the form $ax^3 + bx^2 + cx + d$ where a, b, c and d are integers.

$$(x+2)(x-3) = x^2 - x - 6$$

$$(x+4)(x^2 - x - 6) = x^3 - \underline{x^2} - 6x + \underline{4x^2} - 4x - 24$$
$$= x^3 + 3x^2 - 10x - 24$$

$$a = 1$$
$$b = 3$$
$$c = -10$$
$$d = -24$$

(Total for Question 12 is 3 marks)

- 13 Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6.

n = any number

$2n$ = always even

3 consecutive even numbers = $2n, 2n+2, 2n+4$

$$\text{Sum} = 2n + 2n+2 + 2n+4$$

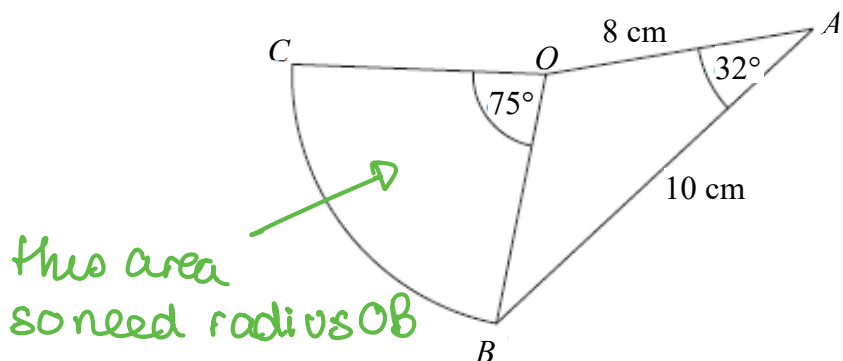
$$= 6n + 6$$

$$= 6(n+1)$$

always a multiple of 6

(Total for Question 13 is 3 marks)

- 14 OAB is a triangle.
 OBC is a sector of a circle, centre O .



Calculate the area of OBC .
 Give your answer correct to 3 significant figures.

$$OB^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 32$$

$$OB = \sqrt{28.31\dots}$$

$$= 5.32\dots$$

$$\text{area } OBC = \frac{75}{360} \times \pi \times 5.32\dots^2$$

$$= 18.5303\dots$$

$$= 18.5 \text{ (3sf.)}$$

.....18.5..... cm^2

(Total for Question 14 is 4 marks)

15 (a) Factorise $p^2 - q^2$

difference of two squares

$$\dots (p+q)(p-q) \dots \quad (1)$$

(b) Show that $3^{60} - 1$ is the product of two consecutive even numbers.

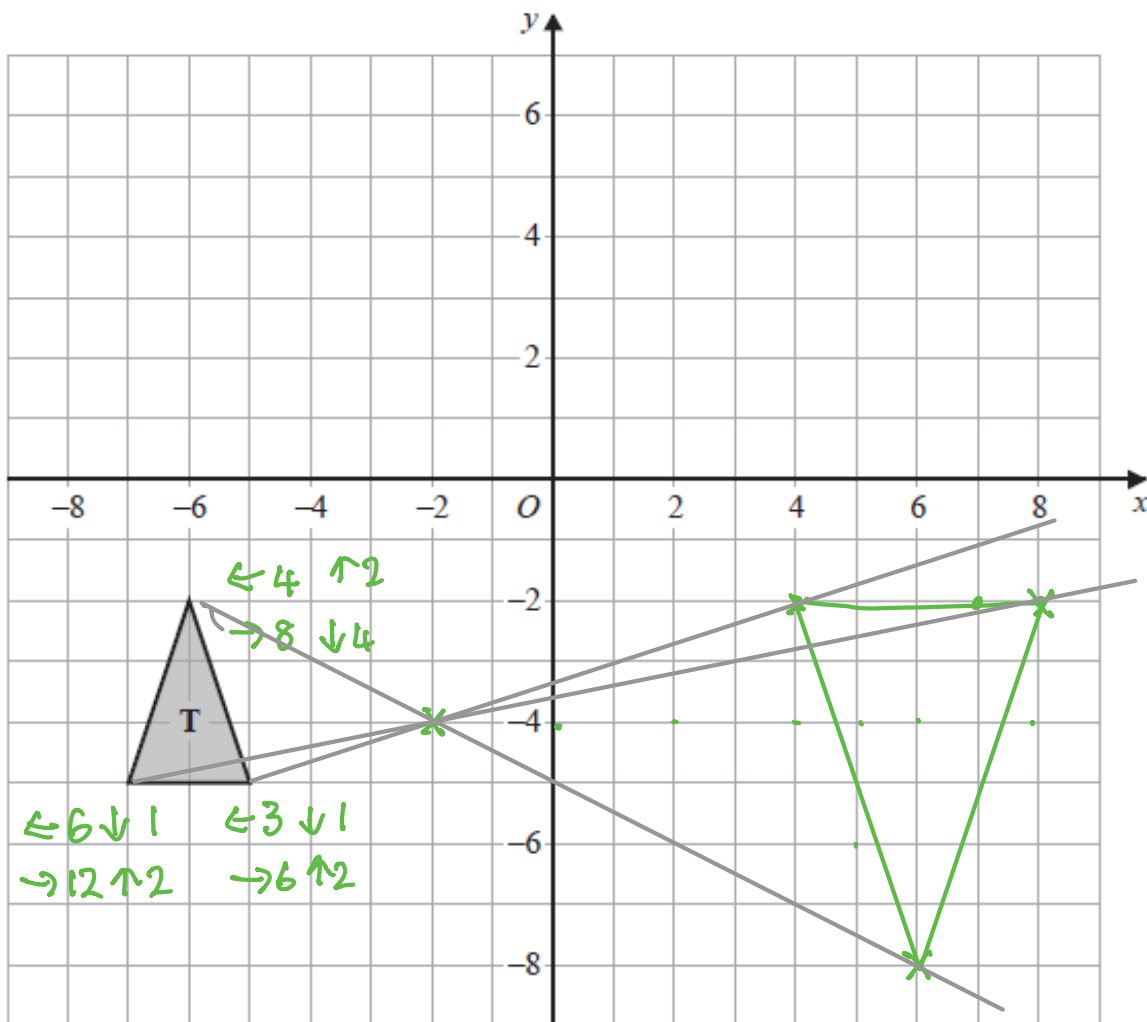
$$\begin{aligned} p^2 - q^2 \\ \text{so } (3^{30})^2 - 1^2 \\ = (3^{30} + 1)(3^{30} - 1) \end{aligned}$$

$$\begin{aligned} 3^{30} = \text{odd} \quad 3^{30} + 1 = \text{even and} \\ 3^{30} - 1 = \text{even} \end{aligned}$$

(2)

(Total for Question 15 is 3 marks)

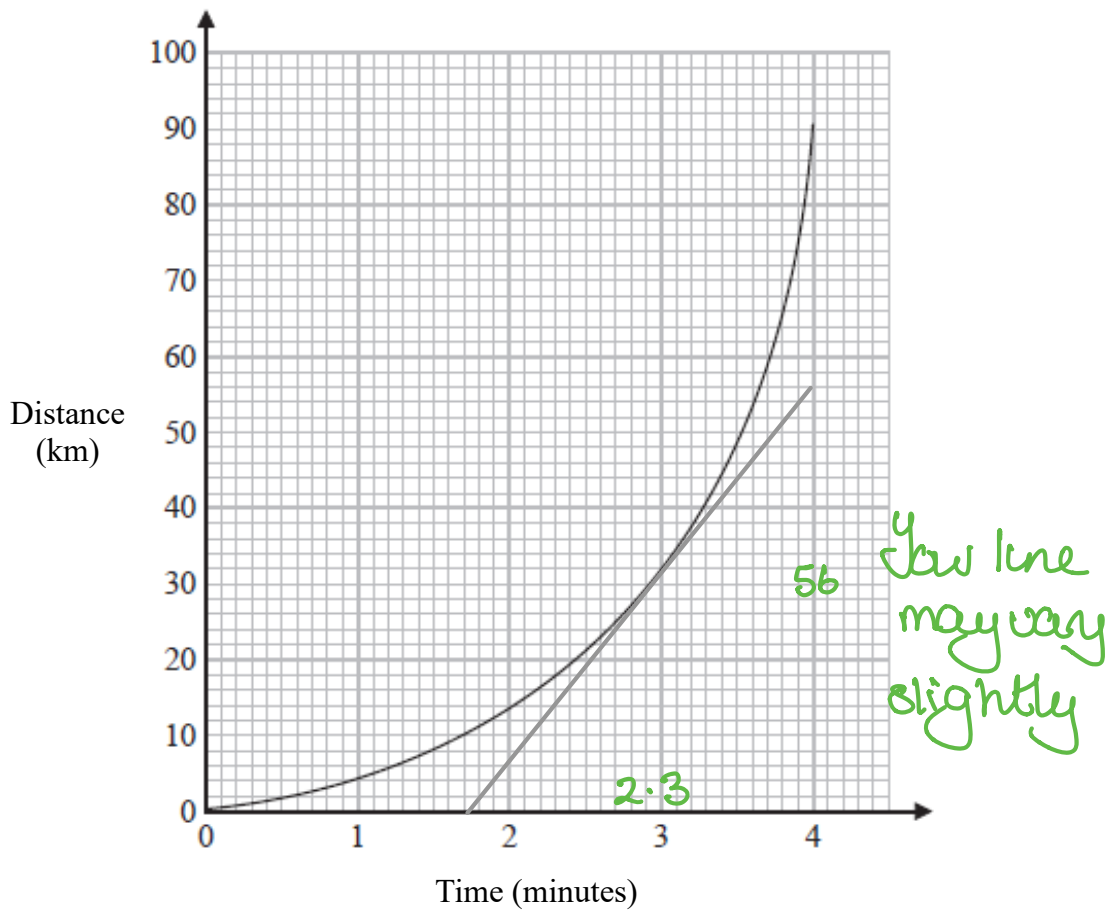
16



On the grid, enlarge triangle **T** by scale factor -2 with centre of enlargement $(-2, -4)$

(Total for Question 16 is 2 marks)

17 Here is a distance-time graph.



- (a) Find an estimate of the gradient of the graph at time 3 minutes.
You must show how you get your answer.

$$\frac{56}{2.3} = 24.347\dots$$

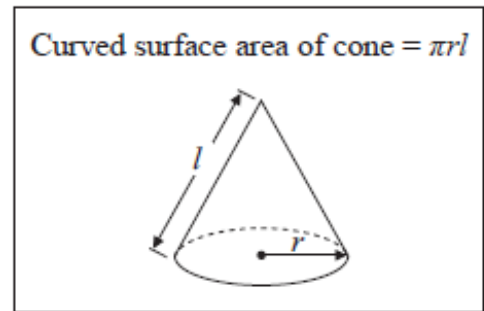
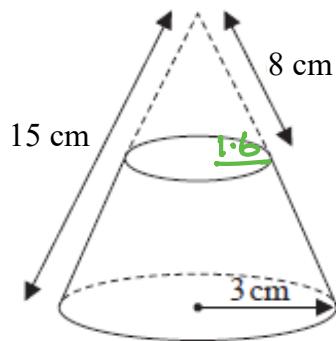
..... 24 (3)

- (b) What does the gradient of the graph represent?

..... Velocity (or Speed) (1)

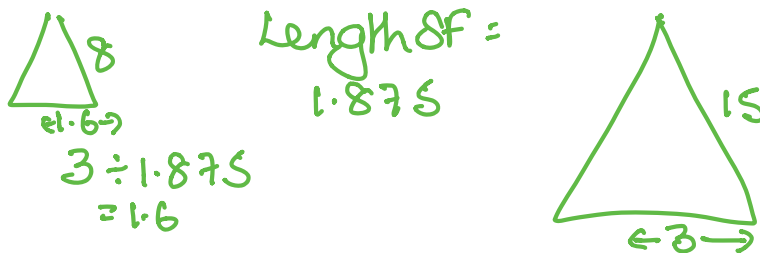
(Total for Question 17 is 4 marks)

18 A solid frustum is made by removing a small cone from a large cone as shown in the diagram.



The slant height of the small cone is 8 cm.
 The slant height of the large cone is 15 cm.
 The radius of the base of the large cone is 3 cm.

Calculate the total surface area of the frustum.
 Give your answer correct to 3 significant figures.



Frustum - curved area.

$$= \pi \times 3 \times 15 - \pi \times 1.6 \times 8$$

$$= 32.2\pi$$

$$\underline{\text{Total}} = 32.2\pi + \pi \times 1.6^2 + \pi \times 3^2$$

$$= 32.2\pi + 2.56\pi + 9\pi$$

$$= 43.76\pi$$

$$= 137.476\dots$$

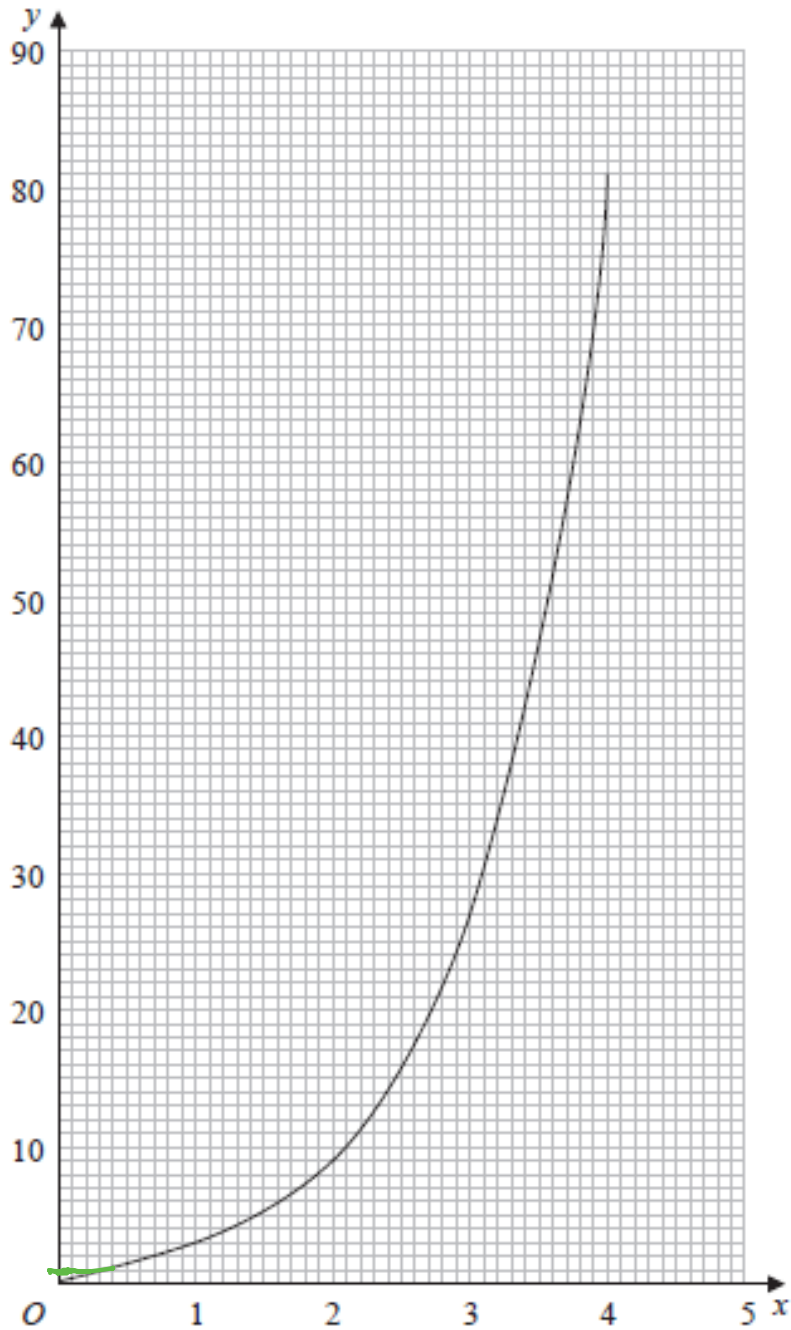
..... 137 (3sf) cm²

(Total for Question 18 is 5 marks)

19 Saira needs to draw the graph of $y = 3^x$ for $0 \leq x \leq 4$

when $x=0$
 $3^0 = 1$

She draws the graph shown on the grid.



Write down one thing Saira has done wrong.

When $x=0$ the graph should go through the y axis at $y=1$ ($3^0=1$)

(Total for Question 19 is 1 mark)

20 Prove algebraically that $0.2\dot{5}\dot{6}$ can be written as $\frac{127}{495}$

$$\begin{array}{r} 100x = 25.656\dots \\ x = 0.256\dots \\ \hline 99x = 25.4 \end{array}$$

$$x = \frac{25.4}{99}$$

$$\frac{25.4}{99} = \frac{254}{990}$$

$$\frac{254}{990} = \frac{127}{495} \text{ as required}$$

(Total for Question 20 is 3 marks)

21 Solve $\frac{1}{x+5} + \frac{4}{2-2x} = 1$

$$2 - 2x + 4(x+5) = (x+5)(2-2x)$$

$$2 - \cancel{2x} + \cancel{4x} + 20 = \cancel{2x} - 2x^2 + 10 - 10x$$

$$2x^2 + 10x + 12 = 0$$

$$2(x^2 + 5x + 6) = 0$$

$$2(x+2)(x+3) = 0$$

$$\text{so } x = -2 \text{ and } x = -3$$

$$\underline{x = -2 \text{ and } x = -3}$$

(Total for Question 21 is 4 marks)

22 Given that the vector $p \begin{pmatrix} 3 \\ 5 \end{pmatrix} + q \begin{pmatrix} 4 \\ 8 \end{pmatrix}$ is parallel to the vector $\begin{pmatrix} 26 \\ 50 \end{pmatrix}$

find an expression for q in terms of p .

$$3p + 4q = 26 \quad (1) \times 2$$

$$5p + 8q = 50 \quad (2)$$

$$6p + 8q = 52 \quad (3)$$

$$(3) - (2) \quad p = 2$$

$$\text{sub in (1)} \quad 6 + 4q = 26$$

$$4q = 20$$

$$q = 5$$

$$\text{so } q = 2.5p$$

$$q = 2.5p$$

(Total for Question 22 is 3 marks)

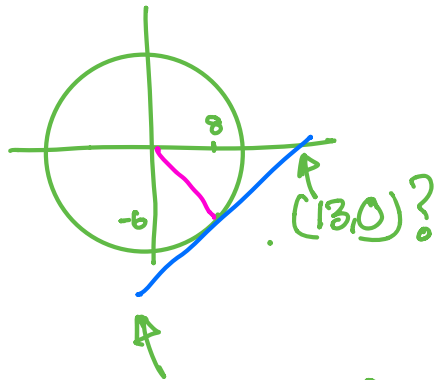
23 A circle has equation $x^2 + y^2 = 100$

The point P with coordinates $(8, -6)$ lies on the circle.

Ayesha says that the tangent to the circle at P crosses the x -axis at the point $(13, 0)$

Is Ayesha correct?

You must show how you get your answer.



$$\text{gradient of radius} = -\frac{6}{8} = -\frac{3}{4}$$

$$\text{gradient of tangent} = \frac{4}{3}$$

$$\text{equation of line} \Rightarrow y = \frac{4}{3}x + c$$

$$\text{passes through } \begin{matrix} (8, -6) \\ x \quad y \end{matrix}$$

$$-6 = \frac{4}{3} \times 8 + c$$

$$c = -6 - \frac{32}{3} = -\frac{18}{3} - \frac{32}{3}$$

$$= -\frac{50}{3}$$

$$\text{so } y = \frac{4}{3}x - \frac{50}{3}$$

$$\text{when } x = 13 \quad y = \frac{52}{3} - \frac{50}{3} = \frac{2}{3}$$

$$\text{so } \left(13, \frac{2}{3}\right) \text{ Ayesha is incorrect}$$

(Total for Question 23 is 4 marks)

24 There is a total of y sweets in a packet.

There are x green sweets and 6 orange sweets in the packet.

The rest of the sweets are yellow.

$$x : y = 1 : 4$$

$$\frac{x}{y} = \frac{1}{4} \quad \text{so } y = 4x$$

Hannah takes at random two sweets from the packet.

Find, in terms of x , an expression for the probability that Hannah takes two sweets of the same colour.

Give your answer as a fraction in the form $\frac{ax^2 + bx + c}{dx^2 + ex}$ where a, b, c, d and e are integers.

G	O	Y	Total
x	6	$y - (x + 6)$	$y = 4x$
		$= 4x - x - 6$	
		$= 3x - 6$	

$P(\text{2 sweets the same colour})$

$$= P(G, G) + P(O, O) + P(Y, Y)$$

$$= \frac{x}{4x} \times \frac{x-1}{4x-1} + \frac{6}{4x} \times \frac{5}{4x-1} + \frac{3x-6}{4x} \times \frac{3x-7}{4x-1}$$

$$= \frac{x(x-1) + 30 + 9x^2 - 39x + 42}{4x(4x-1)}$$

$$= \frac{x^2 - x + 30 + 9x^2 - 39x + 42}{16x^2 - 4x}$$

$$= \frac{10x^2 - 40x + 72}{16x^2 - 4x}$$

(Total for Question 24 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS